

Entwistle Mathematics



Year 12 Extension 1 Mock Exam

General Instructions:

- Reading time – 10 minutes
- Working time – 2 hours
- Write using a black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper.
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section.

Section II – 60 marks

- Attempt Questions 1-14
- Allow about 1 hours and 45 minutes for this section.

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

- 1 The polynomial $P(x) = 4x^3 - 8x^2 - x + k$ is divisible by $(x - 2)$.

What is the value of k ?

(A) 0

(B) 1

(C) 2

(D) 3

- 2 Which of the following is the exact value of $\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)$?

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{3}}{4}$

(D) $\frac{\sqrt{3}}{2}$

- 3 A force described by the vector $\underline{F} = 2\underline{i} + \underline{j}$ is applied to an object on the line $4x - 3y = 0$.

Which of the following is the component of the force F in the direction perpendicular to the line?

- (A) $0.8\underline{i} + 0.6\underline{j}$
- (B) $0.8\underline{i} - 0.6\underline{j}$
- (C) $-0.8\underline{i} + 0.6\underline{j}$
- (D) $-0.8\underline{i} - 0.6\underline{j}$

- 4 Evaluate the following integral

$$\int_0^1 \frac{1}{\sqrt{16 - 4x^2}} dx$$

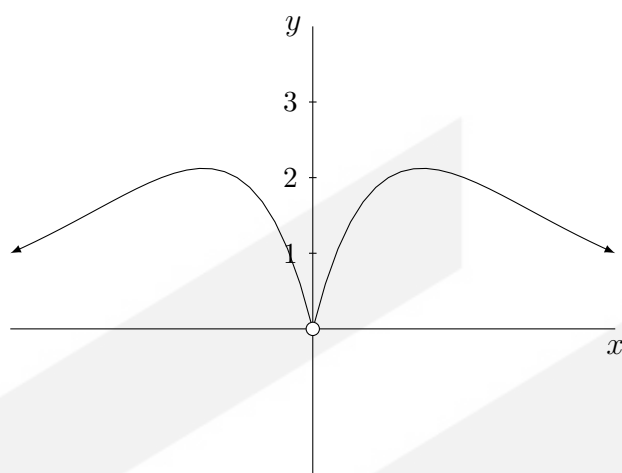
- (A) $\frac{\pi}{16}$
- (B) $\frac{\pi}{12}$
- (C) $\frac{\pi}{6}$
- (D) $\frac{\pi}{4}$

- 5 The letters of the word MATHEMATICS are arranged randomly to form a word.

How many words can be formed if the E is somewhere in between the M's?

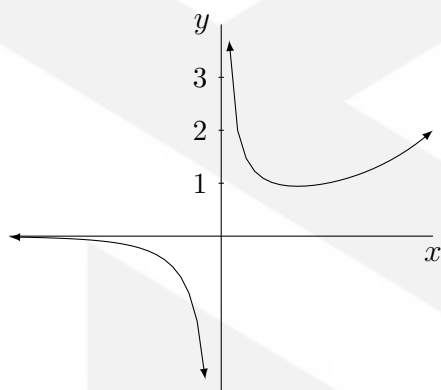
- (A) 1663200
- (B) 2494800
- (C) 3326400
- (D) 4989600

- 6 The graph of $y = \frac{4}{f(|x|)}$ is provided on the diagram below.

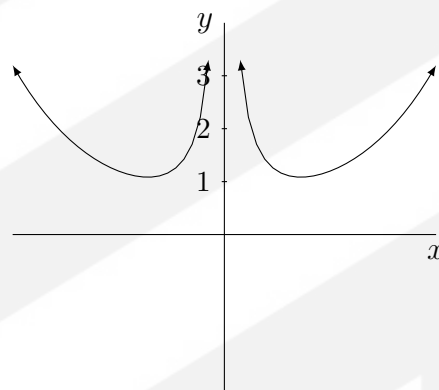


Which of the following is a possible graph for the original function $y = f(x)$?

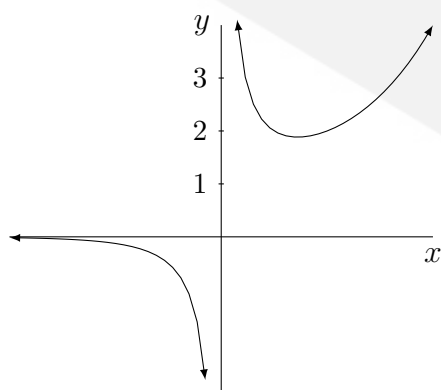
(A)



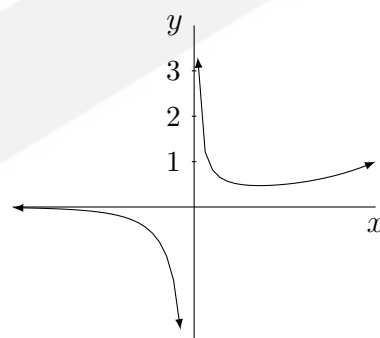
(C)



(B)



(D)



- 7 Let $\vec{a}$ and $\vec{b}$ be non-zero vectors. Suppose the projection of $\vec{a}$ onto $\vec{b}$ is equal to $\frac{1}{2}\vec{b}$. Which of the following vectors must be perpendicular to $\vec{b}$?

(A) $2\vec{a} - \vec{b}$

(B) $\vec{a} + \frac{1}{2}\vec{b}$

(C) $2\vec{a} + \vec{b}$

(D) $\vec{a} - 2\vec{b}$

- 8 Let $f(x)$ is a strictly increasing and differentiable function with domain on all real values of x .

Suppose the graph of $f(x)$ passes through $(2, 5)$ with gradient 3.

Which of the following is impossible?

(A) The graph of $f^{-1}(x)$ has gradient m at $x = 2$ where $m > 3$.

(B) The graph of $f^{-1}(x)$ has gradient $\frac{1}{3}$ at $(5, 2)$.

(C) The graph of $f^{-1}(x)$ passes through a point $(2, b)$ where $b > 2$.

(D) The graph of $f^{-1}(x)$ passes through $(5, 2)$.

- 9** Eight people sit around a circular table.

How many distinct seating arrangements are there such that Alice, Bob and Charlie appear in clockwise order, not necessarily consecutively, and Bob is *not* sitting directly opposite Charlie?

- (A) 840
- (B) 2160
- (C) 2400
- (D) 2520

- 10** Alice tosses a fair coin 30 times and Bob tosses a fair coin 20 times independently.

What is the probability, correct to 3 decimal places, that Alice and Bob obtain the same number of heads?

- (A) 0.022
- (B) 0.042
- (C) 0.079
- (D) 0.118

Section II

60 marks

Attempt Questions 11–14

Allow about 1 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available. In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet.

(a) Simplify $\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right)$, showing all working. 2

(b) By finding a constant R such that $2\sqrt{3}\cos x - 5\sin x = R\cos(x + \alpha)$, find the maximum value of $2\sqrt{3}\cos x - 5\sin x$. 3

(c) Determine the coefficient of x^5 in $\left(2x^2 - \frac{1}{x}\right)^7$. 2

(d) Let u and v be vectors. Prove that 2

$$\frac{|u + v|^2 + |u - v|^2}{2} = |u|^2 + |v|^2$$

(e) Let $f(x) = \sqrt{x + 3} - \sqrt{x - 1}$.

(i) By considering $f'(x)$ or otherwise, explain why $f(x)$ has an inverse function. 1

(ii) *Without* finding the equation of $y = f^{-1}(x)$, find the domain and range of the inverse function $y = f^{-1}(x)$. 3

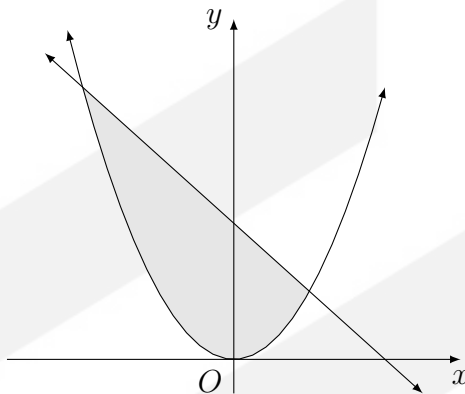
(f) Consider the differential equation $\frac{dy}{dx} = y(2 - y) - x^2$. 2

By considering $\frac{d^2y}{dx^2}$, prove that there are *no* local maxima with a positive x -coordinate for any solution of the differential equation.

End of Question 11

Question 12 (16 marks) Use the Question 12 Writing Booklet.

- (a) Let R be the region bounded by the curve $y = x^2$, $y = 2 - x$ and the x -axis as shown in the diagram below. 4



Find the volume of the solid formed when R is rotated about the x -axis.

- (b) (i) Expand $(1 - \cos^2 \theta)^3$. 1

- (ii) Using the substitution $x = \sin^2 \theta$ and (i), show that 3

$$\int_0^{\frac{3}{4}} \frac{x^3}{\sqrt{1-x}} dx = \frac{289}{2240}$$

- (c) Find the particular solution to the differential equation $(y - 1)\frac{dy}{dx} = xy$ that passes through the point $(-1, e)$. Express your solution in the form $x = f(y)$. 4

- (d) The polynomial $P(x) = x^4 + ax^2 + bx + c$ has a double root $x = \alpha$ and a double root $x = \beta$.

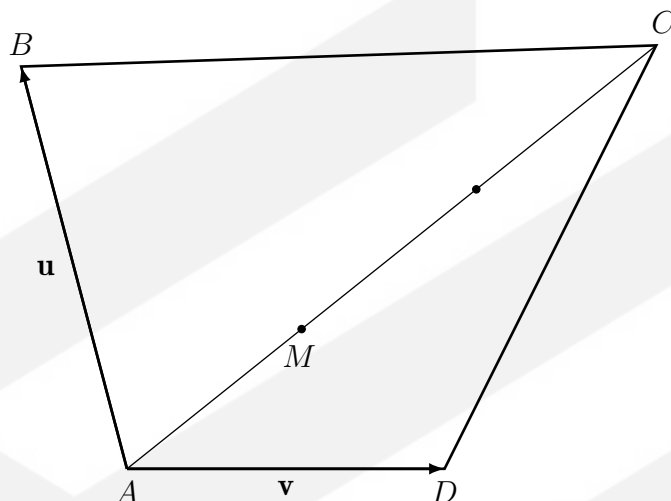
- (i) Show that $a^2 = 36c$. 2

- (ii) Find an expression for b in terms of a . 2

End of Question 12

Question 13 (13 marks) Use the Question 13 Writing Booklet.

- (a) Consider the quadrilateral $ABCD$ with vectors $\overrightarrow{AB} = \mathbf{u}$ and $\overrightarrow{AD} = \mathbf{v}$ where the vectors $\mathbf{u}$ and $\mathbf{v}$ are not parallel. The point M lies on the line AC such that $AM : MC = 1 : 2$.



- (i) Suppose $\lambda_1 \mathbf{u} + \lambda_2 \mathbf{v} = \mu_1 \mathbf{u} + \mu_2 \mathbf{v}$ for constants $\lambda_1, \lambda_2, \mu_1$ and μ_2 . **2**

By considering the fact that $\mathbf{u}$ and $\mathbf{v}$ are *not* parallel, explain why $\lambda_1 = \mu_1$ and $\lambda_2 = \mu_2$.

- (ii) Hence, or otherwise, explain why there exists unique constants a and b such that $\overrightarrow{BC} = a\mathbf{u} + b\mathbf{v}$. **1**

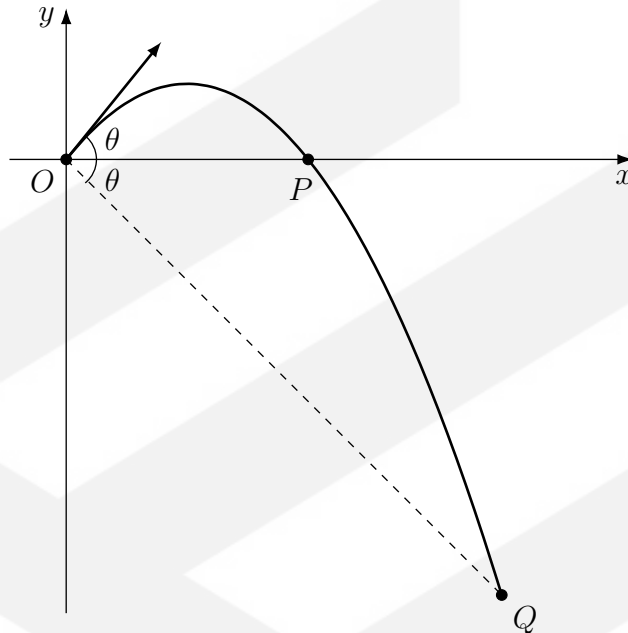
- (iii) Suppose $\overrightarrow{BC} = a\mathbf{u} + b\mathbf{v}$. Show that if M lies on DB then $a + b = 2$. **2**

- (iv) Hence determine the vector $\overrightarrow{AM}$ if the points D, M and B are collinear and $DM : MB = 1 : 3$. **2**

Question 13 continues on page 9

Question 13 (continued)

- (b) A projectile is fired from the origin O with initial speed $V \text{ m s}^{-1}$ at an angle of elevation of θ from the horizontal.



The trajectory passes through the x -axis at point P . The particle is allowed to fall until its angle of depression from the origin is θ .

Assume that the acceleration due to gravity is $g \text{ m s}^{-2}$.

- (i) Show that $x = Vt \cos \theta$ and $y = -\frac{1}{2}gt^2 + Vt \sin \theta$. **2**
- (ii) Find the time taken to reach P , in terms of θ and V . **1**
- (iii) Prove that it takes twice as long to reach Q as it does to reach P . **3**

End of Question 13

Question 14 (16 marks) Use the Question 14 Writing Booklet.

- (a) Suppose Alice flips n coins, where each coin land heads with probability p . The number of heads is some random variable A .

Bob then re-flips all of the A heads, where each coin will now land heads with probability q . The number of heads flipped by Bob is therefore some random variable B .

- (i) Prove that

2

$$\binom{n}{k} \binom{k}{r} = \binom{n}{r} \binom{n-r}{k-r}$$

for non-negative integers $n \geq k \geq r \geq 0$.

- (ii) Show that the probability that Alice flips a heads and Bob flips b heads is given by

2

$$P(\{A = a\} \cap \{B = b\}) = \binom{n}{b} (pq)^b \binom{n-b}{a-b} [p(1-q)]^a (1-p)^{n-a}$$

where $n \geq a \geq b \geq 0$ and a, b are integers.

- (iii) Hence show that

2

$$P(B = b) = \binom{n}{b} (pq)^b (1-pq)^{n-b}.$$

- (iv) Explain the result of (iii) intuitively.

1

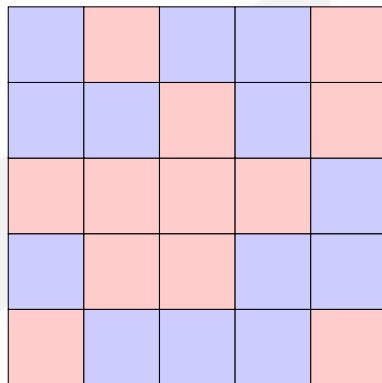
- (v) Suppose Alice flipped 100 heads. By considering a normal approximation, determine the value(s) of q for which Bob will flip fewer than 50 heads at least 84% of the time.

3

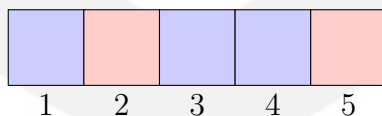
Question 14 continues on page 11

Question 14 (continued)

- (b) Consider a 5×5 grid consisting of 25 cells. Each cell is shaded either red or blue as seen in the diagram below.



In each row, a ‘ticket’ is generated whenever there is a pair of same coloured-cells in a row. Each ticket records the colour of the pair, along with a tuple (a, b) ($a < b$) which identifies the two column indices for the matching cells in that row. For example the ticket $(\mathcal{B}, (1, 4))$ is generated whenever any row has two matching blue cells in columns 1 and 4. The top row of the diagram provided is



which generates the set of four tickets given below:

$$\{(\mathcal{B}, (1, 3)), (\mathcal{B}, (1, 4)), (\mathcal{B}, (3, 4)), (\mathcal{R}, (2, 5))\}.$$

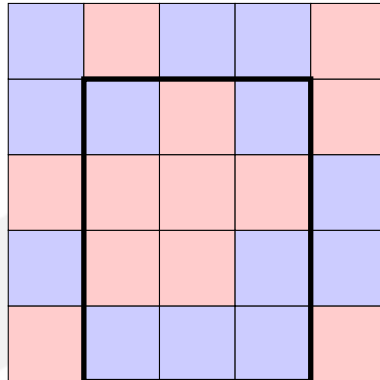
Note that tickets are not determined uniquely by the rows, so there could be two rows that have common tickets. For example the top two rows in the example grid above both generated the ticket $(\mathcal{B}, (1, 4))$.

- (i) Let r be the number of red cells in a particular row. 2
- Show that the number of tickets will be $r^2 - 5r + 10$ and deduce that this is at least 4.
- (ii) Explain why the maximum number of unique tickets for the grid is 20. 1

Question 14 continues on page 12

Question 14 (continued)

In the example grid below, there is a rectangular region of cells where the corner vertices all share the same colour. (There may be a few others).



- (iii) By considering the previous parts, demonstrate that there will always exist a rectangular region of cells where the corner cells are all the same colour. 3

2019–2026 We watched you soar, we watched you fly, Through countless proofs and questions why. Though NESA says your time is through, There'll always be a hole for you.



END OF AN ERA (and this EXAM)